

The SAT Initiative

G7 Topic Breakdown

SOL - Geometry

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Topic: Prove two triangles are similar, using algebraic and coordinate methods as well as deductive proofs.

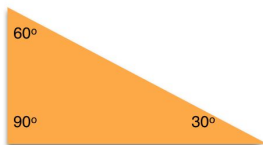
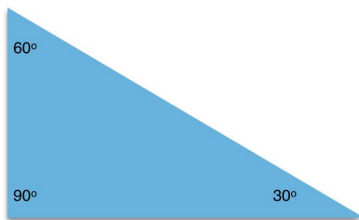
Two triangles are **similar** when:

All their angles are equal

$$60^\circ = 60^\circ$$

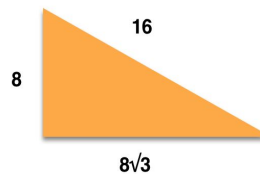
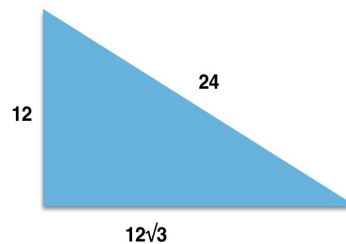
$$30^\circ = 30^\circ$$

$$90^\circ = 90^\circ$$



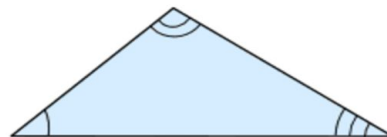
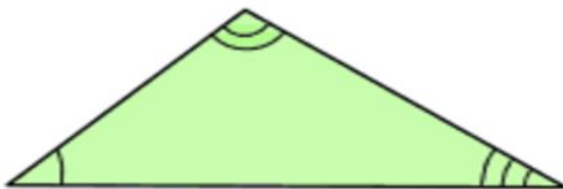
Their corresponding sides have the same ratio

$$\frac{24}{16} = \frac{12}{8} = \frac{12\sqrt{3}}{8\sqrt{3}} = \frac{3}{2}$$



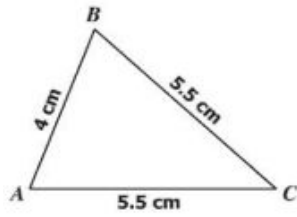
Proving two triangles are similar:

- ★ **Angle-Angle (AA) Similarity:** If two angles in a triangle are congruent to the two corresponding angles in a second triangle, then the two triangles are similar.

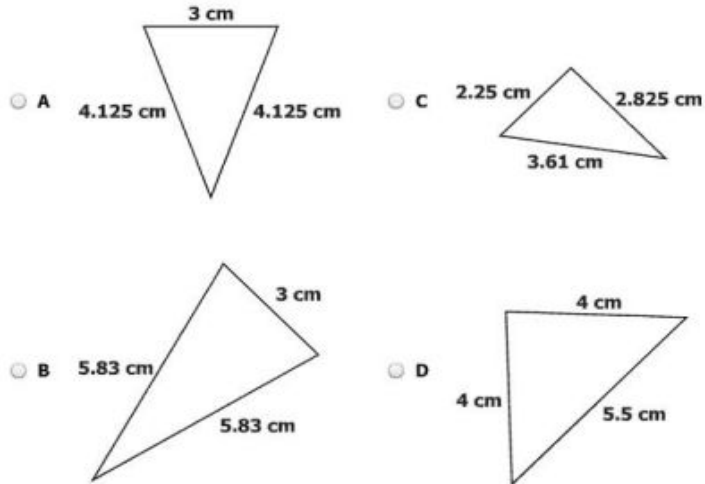


★ Side-Side-Side (SSS) Similarity:

Look at this triangle.



Which triangle is similar to the given triangle?



Determine the order of sides, from smallest to largest:

$$AB < AC = BC \quad (4 \text{ cm} < 5.5 \text{ cm} = 5.5 \text{ cm})$$

Notice that in the ABC triangle, $\frac{AC}{BC} = 1$. We could eliminate the choice C, which does not have two equal sides. Try finding the ratio between each side of the multiple choice triangle and the side of the original triangle.

$$\text{A) } \frac{3}{4} = \frac{4.125}{5.5} = \frac{4.125}{5.5} = 0.75$$

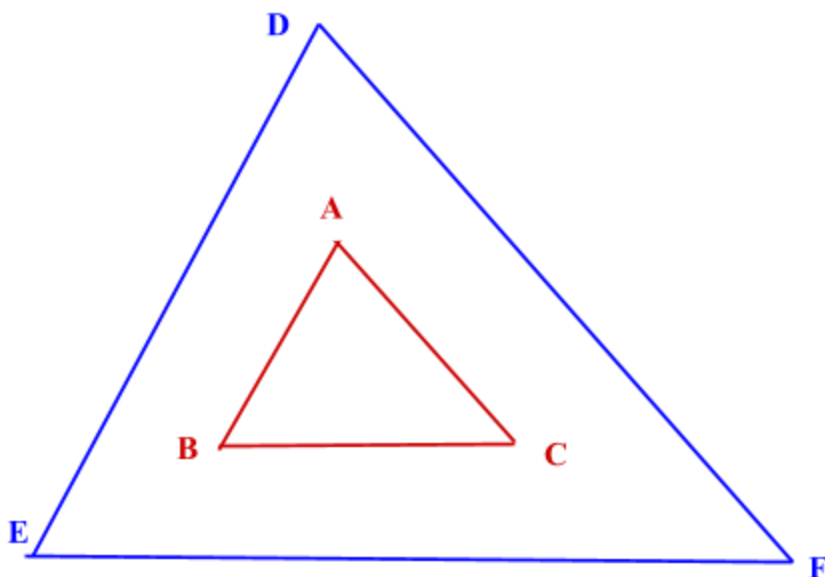
$$\text{B) } \frac{3}{4} = 0.75 < \frac{5.83}{5.5} = \frac{5.83}{5.5} = 1.06$$

$$\text{D) } \frac{5.5}{4} = 1.375 > \frac{4}{5.5} = \frac{4}{5.5} = 0$$

The answer would be A, because the corresponding sides have the same ratio.

Applying Distance Formula

Let the vertices of triangles **ABC** and **DEF** defined by the coordinates: A(0, 3), B(-1,1), C(1,1), D(0,4), E(-2,0), and F(2,0). Show that the two triangles are similar.



Use the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$\diamond \mathbf{AB} = \sqrt{(-1 - 0)^2 + (1 - 3)^2} = \sqrt{5}$$

$$\diamond \mathbf{BC} = \sqrt{(1 - (-1))^2 + (1 - 1)^2} = 2$$

$$\diamond \mathbf{AC} = \sqrt{(1 - 0)^2 + (1 - 3)^2} = \sqrt{5}$$

$$\diamond \mathbf{DE} = \sqrt{(-2 - 0)^2 + (0 - 4)^2} = 2\sqrt{5}$$

$$\diamond \mathbf{EF} = \sqrt{(2 - (-2))^2 + (0 - 0)^2} = 4$$

$$\diamond \mathbf{DF} = \sqrt{(2 - 0)^2 + (0 - 4)^2} = 2\sqrt{5}$$

We have:

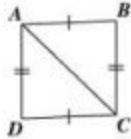
$$\frac{\mathbf{DE}}{\mathbf{AB}} = \frac{\mathbf{EF}}{\mathbf{BC}} = \frac{\mathbf{DF}}{\mathbf{AC}} = 2$$

So **ABC** and **DEF** are similar.

SOL Practice Problems

Given: Figure $ABCD$ with diagonal AC

$$\overline{AB} \cong \overline{DC}; \overline{BC} \cong \overline{AD}$$

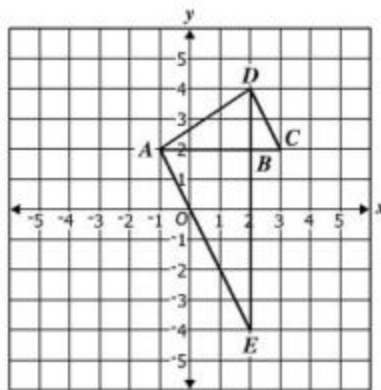


Complete the proof of $\triangle ABC \cong \triangle CDA$ by selecting the reasons for the last two statements.

Statements	Reasons
$\overline{AB} \cong \overline{DC}; \overline{BC} \cong \overline{AD}$	Given
$\overline{AC} \cong \overline{AC}$	
$\triangle ABC \cong \triangle CDA$	

Definition of congruent triangles
Reflexive property
Side-Angle-Side (SAS) Theorem
Side-Side-Side (SSS) Theorem
Hypotenuse-Leg (HL) Theorem
Given

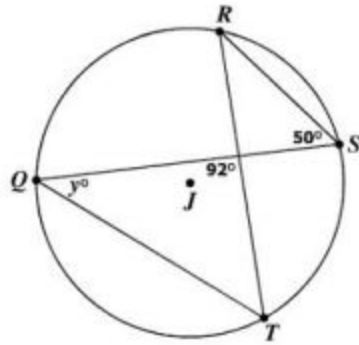
Three triangles that do not overlap are shown on the coordinate grid. The coordinates of all vertices are integers.



Which statement is true?

- A $\triangle ABD \sim \triangle EBA$
- B $\triangle ABD \sim \triangle DBC$
- C $\triangle CBD \sim \triangle ABE$
- D $\triangle CBD \sim \triangle EBA$

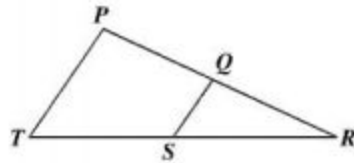
Given: Circle J



What is the value of y ?

- A 38
- B 50
- C 88
- D 92

Given: Q lies on \overline{PR} and S lies on \overline{RT}



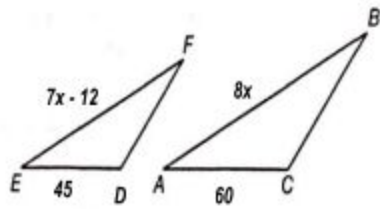
Which condition proves $\triangle PRT \sim \triangle QRS$?

- A $\angle PQS \cong \angle TSQ$
- B $\angle PTR \cong \angle TPR$
- C $\frac{QS}{PT} = \frac{QR}{SR}$
- D $\frac{QR}{PR} = \frac{SR}{TR}$

Additional Practice Problems

G.7 Geometry

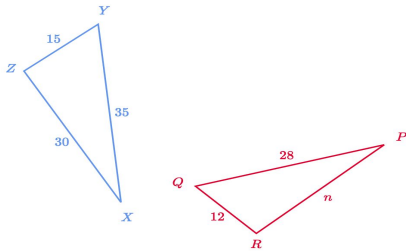
Question 1:



For what value of x will triangle EFD be similar to triangle ABC ?

- A) 4
- B) 5
- C) 6
- D) 7

Question 4:

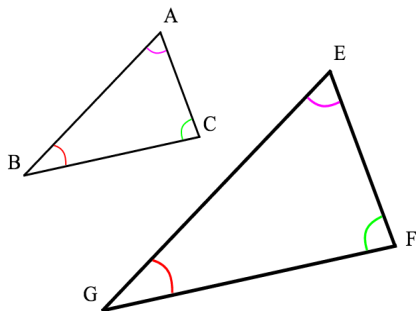


Triangle XYZ is similar to triangle PQR . Find n .

- A) 18
- B) 20
- C) 22
- D) 24

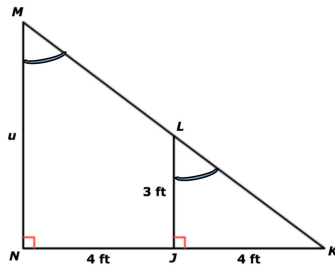
Question 7:

Triangles ABC and EGF are similar. If $\angle A = 60^\circ$, and $\angle G = 40^\circ$, what is the value of $\angle F$?



- A) 60
- B) 80
- C) 120
- D) 130

Question 2:



MN is parallel to LJ . Find u :

- A) 9
- B) 8
- C) 6
- D) 3

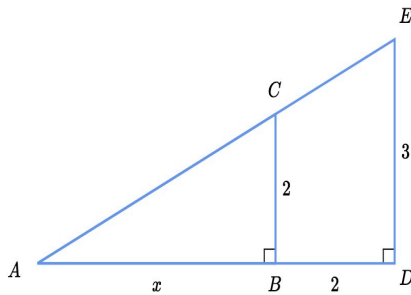
Question 5:

Let the vertices of triangle ABC be defined by point A at $(-1,1)$, point B at $(2,1)$ and point C at $(0,-2)$. Let the vertices of triangle DEF be defined by point D at $(-3,3)$, point E at $(6,3)$ and point F at $(0,-6)$. Are these two triangles similar?

- A) Yes
- B) No
- C) Not enough information

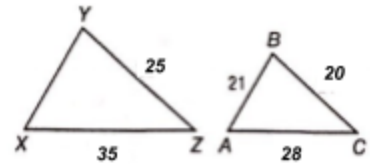
Question 8:

Solve for x .



- A) 4
- B) 6
- C) 8
- D) 10

Question 3:

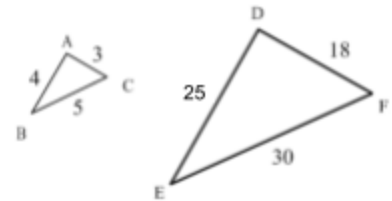


In addition to the figure, what other information would be necessary to prove that triangle XYZ is similar to triangle ABC :

- A) $YX = 17$
- B) $\angle A = \angle Y$
- C) $\angle C = \angle Z$
- D) $\frac{YX}{BC} = 2$

Question 6:

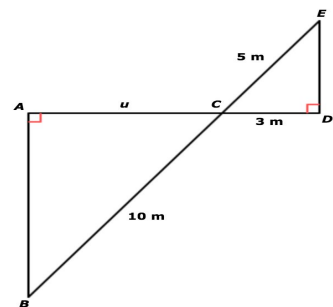
Are these two triangles similar?



- A) Yes
- B) No
- C) Not enough information

Question 9:

In the figure below, triangle ABC is similar to triangle DEC . Find u .



- A) 3
- B) 4
- C) 5
- D) 6

Answer Key: Practice Problems

G.7 Geometry

1.	B
2.	C
3.	C
4.	D
5.	A
6.	B
7.	B
8.	A
9.	D