

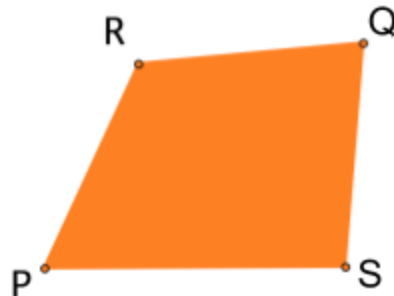
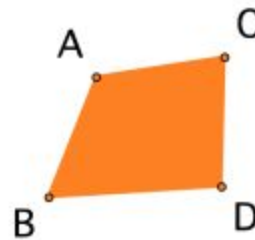
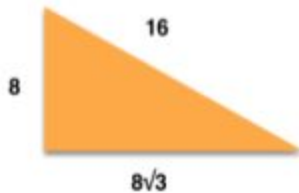
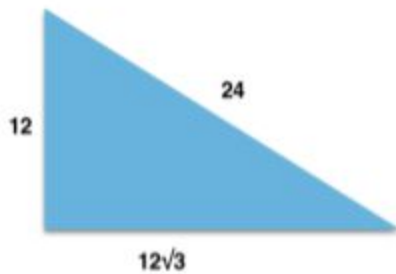
The SAT Initiative

G14 Topic Breakdown
SOL - Geometry
Tracy Nguyen

Topic: Use similar geometric objects in 2D or 3D to compare side lengths, perimeters, areas and volumes.

Geometrically similar: Two objects have the same shape, not necessarily the same size. Two polygons are similar if and only if their corresponding **angles** are **congruent**, and (lengths of) corresponding **sides** are **proportional**.

$$\frac{24}{16} = \frac{12}{8} = \frac{12\sqrt{3}}{8\sqrt{3}} = \frac{3}{2}$$



Scale factor: The ratio of corresponding sides of similar polygons. In the example above, two similar triangles have the scale factor of $\frac{3}{2}$.

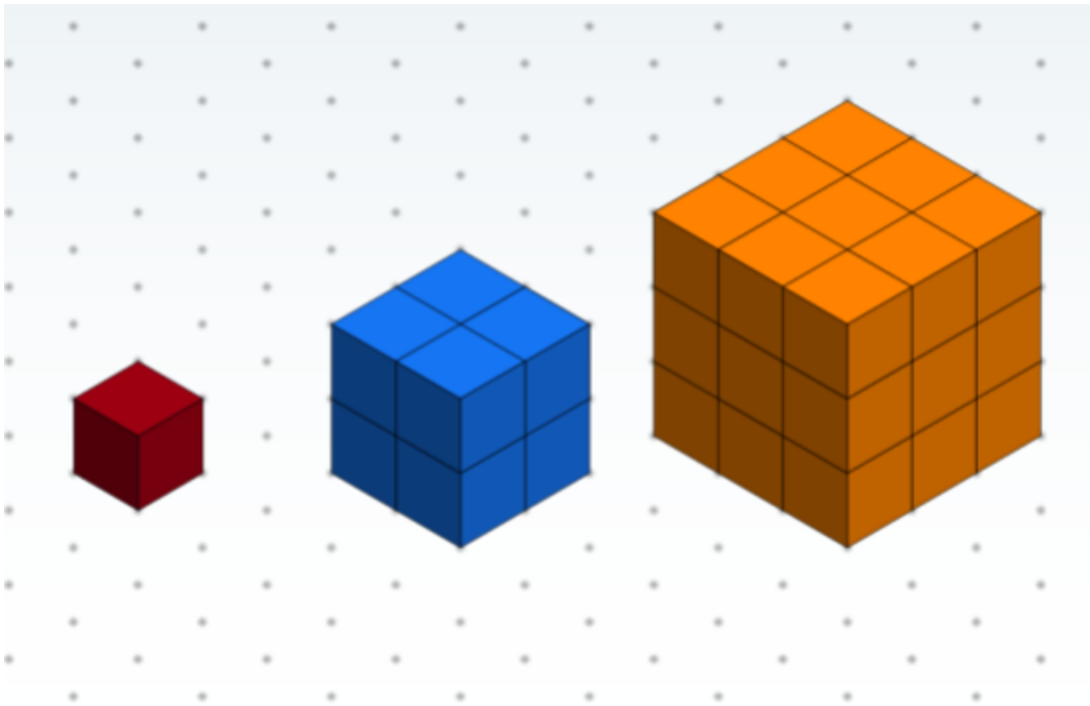
If $AB = 8$, $AC = 6$, $CD = 6$, $BD = 10$, and $RP = 16$, find RQ , QS , SP . What is the scale factor?

$$\frac{RP}{AB} = \frac{16}{8} = 2 = \frac{RQ}{AC} = \frac{QS}{CD} = \frac{SP}{BD}$$

$\Rightarrow RQ = 2 \cdot AC = 12$; $QS = 2 \cdot CD = 12$; $SP = 2 \cdot 10 = 20$

The scale factor for two similar polygons $ABDC \sim RPSQ$ is 2.

Compare ratios between side lengths, perimeters, areas, and volumes



	Red cube	Blue cube	Orange cube	Ratio between Red and Blue	Ratio between Red and Orange	Ratio between Blue and Orange
Side (x)	1	2	3	$\frac{2}{1} = 2$	$\frac{3}{1} = 3$	$\frac{3}{2}$
Surface area ($6x^2$)	6	24	54	$\frac{24}{6} = 4 = 2^2$	$\frac{54}{6} = 9 = 3^2$	$\frac{54}{24} = \frac{9}{4} = \left(\frac{3}{2}\right)^2$
Perimeter ($12x$)	12	24	36	$\frac{24}{12} = 2$	$\frac{36}{12} = 3$	$\frac{36}{24} = \frac{3}{2}$
Volume (x^3)	1	8	27	$\frac{8}{1} = 8 = 2^3$	$\frac{27}{1} = 27 = 3^3$	$\frac{27}{8} = \left(\frac{3}{2}\right)^3$

We notice that for a cube, when the side is doubled from x to $2x$.

❖ The surface area would change from $6x^2$ to $6(2x)^2 = 6 \cdot 4 \cdot x^2 = 24x^2$.

- ❖ The perimeter would change from $12x$ to $12(2x) = 24x$.
- ❖ The volume would change from x^3 to $(2x)^3 = 8x^3$.

To determine how changes in one or more dimensions affect other derived measures (perimeter, area, total surface area, and volume) of an object, follow how the dimension is changed by the formula used to calculate those measures.

Reference formula to find surface area of common geometric figures

Geometric figure	Surface area
Square	side ²
Rectangle	length × width
Parallelogram	base × height
Triangle	base × height / 2
Trapezoid	height × (base1 + base2) / 2
Circle	pi × radius ²
Ellipse	pi × radius1 × radius2
Cube (surface)	6 × side ²
Sphere (surface)	4 × pi × radius ²
Cylinder (surface of side)	perimeter of circle × height
	2 × pi × radius × height
Cylinder (whole surface)	Areas of top and bottom circles + Area of the side
	2(pi × radius ²) + 2 × pi × radius × height
Cone (surface)	pi × radius × side

Reference formula to find volume of common geometric figures

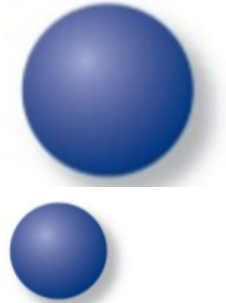
Cube	side ³
Rectangular Prism	side1 × side2 × side3
Sphere	(4/3) × pi × radius ³
Ellipsoid	(4/3) × pi × radius1 × radius2 × radius3
Cylinder	pi × radius ² × height

Cone	$(1/3) \times \pi \times \text{radius}^2 \times \text{height}$
Pyramid	$(1/3) \times (\text{base area}) \times \text{height}$

Practice Problems

The volumes of two spheres are in a ratio of 1:8. What is the ratio of their radii?

- A 1:512
- B 1:64
- C 1:4
- D 1:2



Refer to the table, the formula for volumes of a sphere is $(4/3) \times \pi \times \text{radius}^3$

$$\frac{\text{Volume of sphere 1}}{\text{Volume of sphere 2}} = \frac{(4/3) \times \pi \times (r_1)^3}{(4/3) \times \pi \times (r_2)^3} = \frac{(r_1)^3}{(r_2)^3} = \frac{1}{8}$$

I

So we have:

$$\frac{(r_1)^3}{(r_2)^3} = \left(\frac{r_1}{r_2} \right)^3 = \frac{1}{8} \Rightarrow \frac{r_1}{r_2} = \sqrt[3]{\frac{1}{8}} = \frac{1}{2}$$

The ratio of their radii is thus 1:2, or the answer is D.

The volume of a cube is 64 cubic centimeters. What is the surface area of the cube?

- A 16 cm²
- B 96 cm²
- C 256 cm²
- D 384 cm²

The formula for the volume of a cube is side^3 so if the volume is 64, the length of the side is $\sqrt[3]{64} = 4$. The surface area of the cube is then $6 \times \text{side}^2 = 6 \times 4^2 = 6 \times 16 = 96 \text{ cm}^2$.

Real world practice Problems

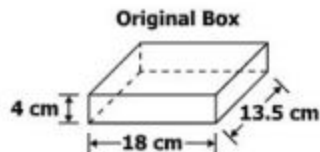
A company is creating a new cylindrical container to replace its original cylindrical container.

- The new container will have 4 times the volume of the original container.
- The height of the new container will remain the same as the height of the original container.

The length of the radius of the new container will be —

- A 2 times the length of the radius of the original container
- B 4 times the length of the radius of the original container
- C 8 times the length of the radius of the original container
- D 16 times the length of the radius of the original container

A cell phone box in the shape of a rectangular prism is shown. The height of the box is 4 cm.



The height of the original box will be increased by 3.5 centimeters so a new instruction manual and an extra battery can be included. Which is closest to the total surface area of the new box?

- A 479 cm²
- B 707 cm²
- C 738 cm²
- D 959 cm²

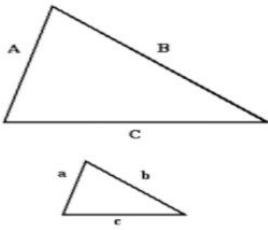
Additional Practice Problems

G.14 Geometry

Question 1:

In the displayed triangles, the lengths of the sides are: $A = 48$ mm, $B = 81$ mm, $C = 66$ mm, and $a = 16$ mm. Find sides b and c ,

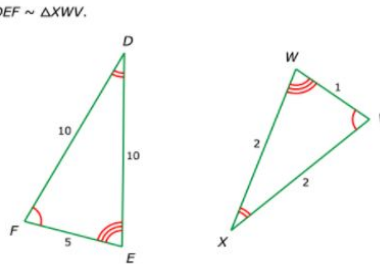
- A) $b = 28$,
 $c = 19$
- B) $b = 27$,
 $c = 22$
- C) $b = 29$,
 $c = 22$
- D) $b = 27$,
 $c = 20$



Question 2:

What is the scale factor?

- A) 3
- B) 5
- C) 7
- D) 9



Question 3:

A cube has side 2. If the side is tripled, what is the volume of this cube now?

- A) 8
- B) 64
- C) 256
- D) 512

Question 4:

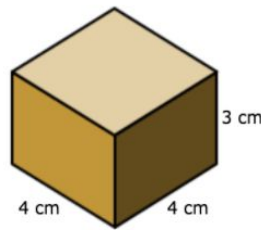
A picture of dimensions 12×20 cm is enlarged so that the width is now 15cm. How long is this picture now?

- A) 20
- B) 22
- C) 25
- D) 28

Question 5:

After doubling the side length 3 of this rectangular prism, what is the ratio between the volume of the original prism and that of the scaled one?

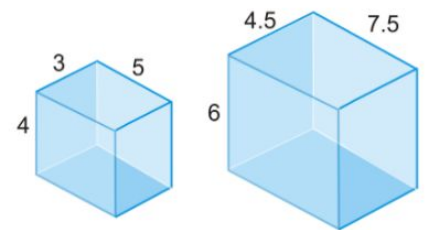
- A) 1:2
- B) 1:3
- C) 1:4
- D) 1:6



Question 6:

Are these two prisms similar?

- A) Yes
- B) No
- C) Not enough information



Question 7:

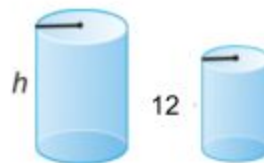
Two spheres have radii ratio of 4:3. What is the ratio of their volumes?

- A) 4:3
- B) 16:9
- C) 64:27
- D) 64:9

Question 8:

If the ratio of the areas of these cylinder is 16:25, what is the height of the taller one?

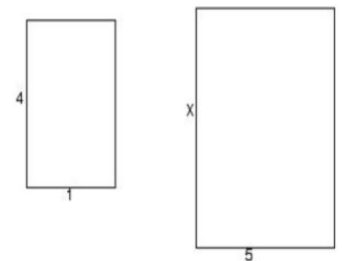
- A) 15
- B) 18
- C) 20
- D) 24



Question 9:

Find the missing side x , knowing that these rectangles are similar.

- A) 10
- B) 12
- C) 16
- D) 20



Answer Key: Practice Problems

G.14 Geometry

1.	B
2.	B
3.	C
4.	C
5.	A
6.	A
7.	C
8.	A
9.	D