The SAT Initiative

G13 Topic Breakdown

SOL - Geometry

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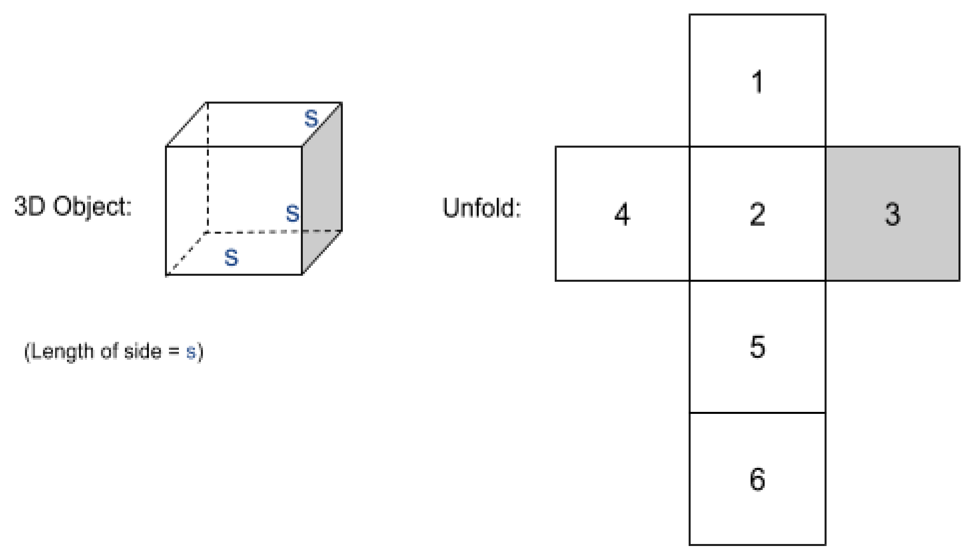
The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

Definitions:

Surface Area: The sum of all of the areas of the sides of an object (units = Distance^2)

Volume: The amount of space inside an object (units = Distance^3)

EXAMPLE:



Surface area of this object= (Area of side 1) + (Area of side 2) + (Area of side 3) + (Area of side 4) + (Area of side 5) + (Area of side 6)

OR

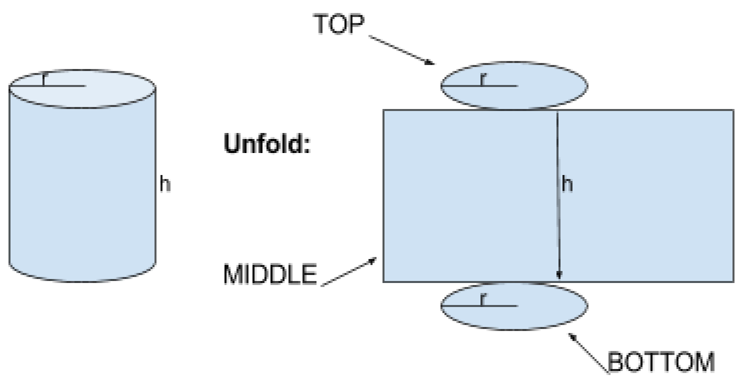
6* (Area of one side)

Volume of this object = s x s x s

Essential Knowledge and Skills
The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to:

1) *Find the total surface area of cylinders, prisms, pyramids, cones, and spheres, using the appropriate formulas.*
2) *Calculate the volume of cylinders, prisms, pyramids, cones, and spheres, using the appropriate formulas.*
3) *Solve problems, including real-world problems, involving total surface area and volume of cylinders, prisms, pyramids, cones, and spheres as well as combinations of three-dimensional figures*

CYLINDERS

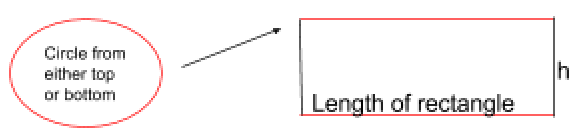


r = radius
h= height

*To find the surface area of the **cylinder**, add the areas of each of the surfaces. For a cylinder, there is a top, middle, and bottom. The top and bottom are both circles with the same dimensions, and the middle is a rectangle.

Area of each circle = $\pi * r^2$
Area of rectangle = length * width
Length of rectangle = (circumference of circle) = $2\pi r$

Circumference of circle = Length of rectangle

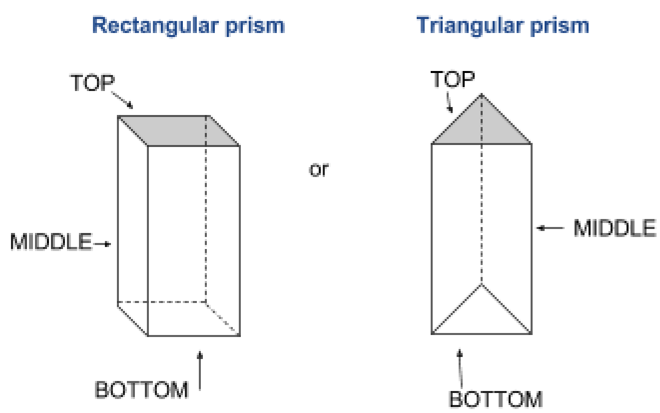


Width of rectangle = h
Area of rectangle = $2\pi r$ *h
Surface area of cylinder = Area of both circles + Area of rectangle
Surface area of cylinder = $2\pi r$ *h + $2\pi r^2$ = $2\pi r$ (h+r)

Volume of cylinder = (area of base) * height = πr^2 *h

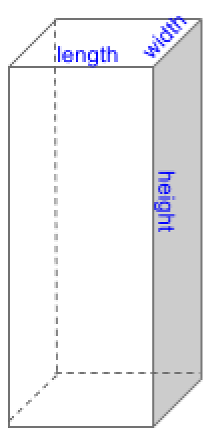
PRISMS

A **prism** is an object with equal ends and a uniform middle:



***TO FIND THE SURFACE AREA OF A PRISM, ADD TOGETHER THE AREAS OF EACH SURFACE**
***TO FIND THE VOLUME OF A PRISM, MULTIPLY THE AREA OF THE BASE BY THE HEIGHT OF THE PRISM**

RECTANGULAR PRISM



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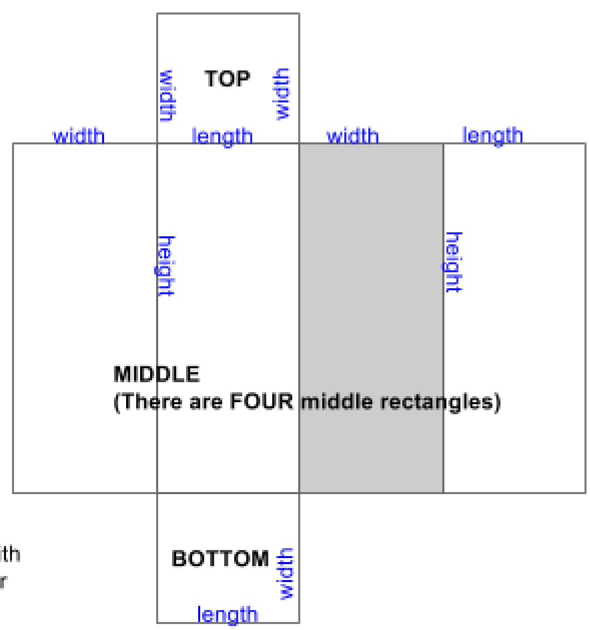


For a rectangular prism, there is a top, middle, and bottom. The top and bottom are both quadrilaterals with the same dimensions. The middle is composed of four rectangles.

Surface area = (sum of all areas of **MIDDLE** rectangles) +(area of **TOP**) + (area of **BOTTOM**)

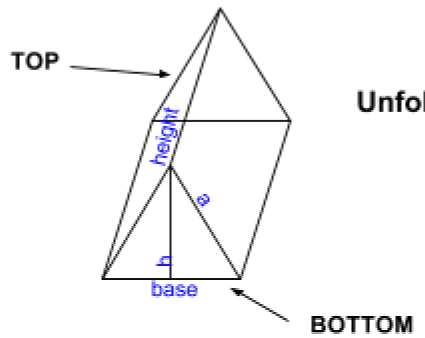
*Area of **TOP** = Area of **BOTTOM** = length x width

*Area of **MIDDLE** rectangles = (length x height) + (length x height) + (width x height) + (width x height)

Surface area of rectangular prism = 2*(length x height) + 2*(width x height) + 2*(length x width)

Volume of rectangular prism = (area of base) * height = length x width x height

TRIANGULAR PRISM



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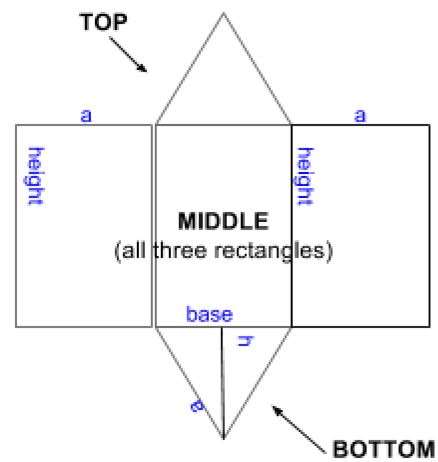

For a triangular prism, there is a top, middle, and bottom. The top and bottom are both triangles with the same dimensions. The middle is composed of three Rectangles, two with the same dimension and one with different dimensions. If the triangle is equilateral, all three rectangles will have the same dimension

Surface area = (sum of areas of **MIDDLE** rectangles) +(area of **TOP**) + (area of **BOTTOM**)

*Area of **TOP** = Area of **BOTTOM** = (½)*(base x h)

*Area of **MIDDLE** rectangles = (height x a) + (base x height) + (height x a)

Surface area of triangular prism = 2*(height x a) + (base x height) + (base x h)

Volume of triangular prism = (area of base) * height = (½)*(base x h)* height

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PYRAMIDS

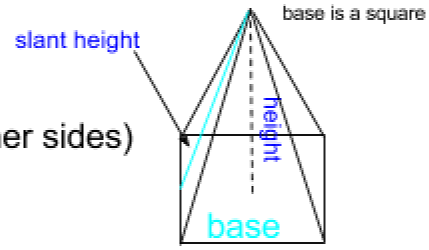
A **pyramid** is a 3D object with either a triangle, square, pentagon, etc, as a base and triangular sides that meet to a point at the top (this point is called the apex).

ALL PYRAMIDS:

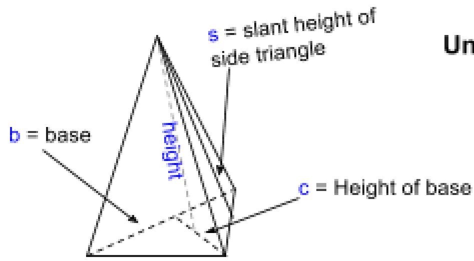
If the base has all sides of equal length,
Surface area = (area of base) + ½(perimeter of base)*(slant height)

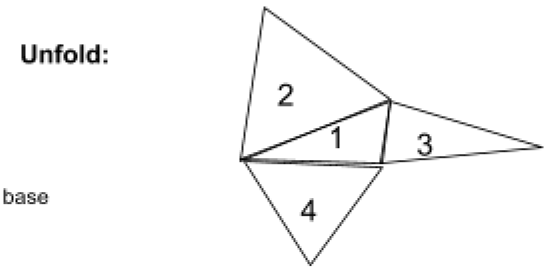
If base has sides of different lengths,
Surface area = (area of base) + (lateral area)
= (area of base) + (sum of area of all other sides)

Volume = ⅓ (area of base) * (height)



Triangular pyramid



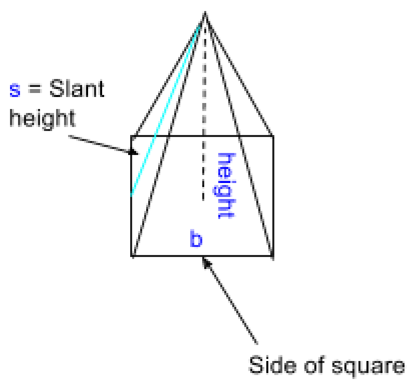


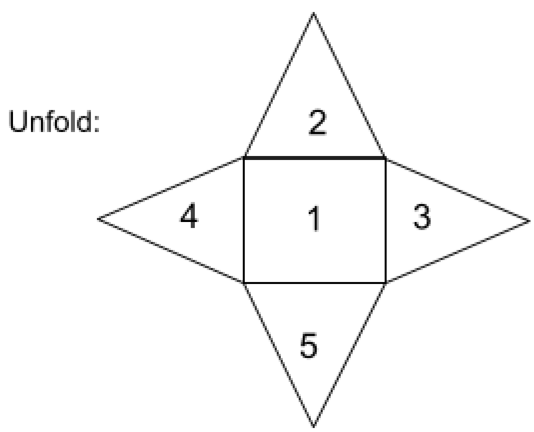
For a triangular pyramid, there is a triangular base and three triangular sides that meet at the apex.
If the triangle is an equilateral triangle, the formula for surface area that can be used is:
Surface area = (base area)+ ½ (perimeter of base) x (slant height)
Surface area = ½ (b x c) + ½ (perimeter of base) x s

If the triangle is not equilateral, add all the areas of each side:
Surface area = (area of base) + (Lateral area)
Surface area = (area of base) + (area of triangle 2)+(area of triangle 3) + (area of triangle 4)

Volume of triangular pyramid = ⅓ (area of base) * height
Volume = ⅓(½ b x c) * height

Square pyramid





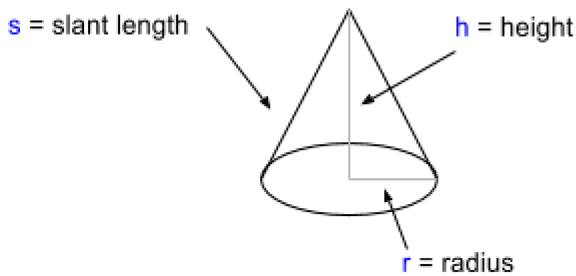
For a square pyramid, there is a square base and four equal triangular sides that meet at the apex.

Surface area = (base area)+ ½ (perimeter of base) x (slant height)
Surface area = (b x b) + ½ (4b) x s

Volume of square pyramid = ⅓ (area of base) * height
Volume = ⅓(b x b) * height

CONES

A **cone** is a 3D object with a circular base and a point at the top.

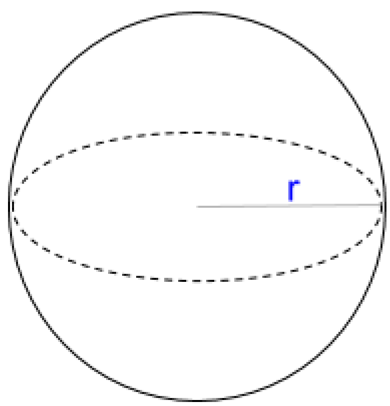


Surface area = (area of cone) + (area of base)
Surface area = $(\pi rs) + (\pi r^2)$

Volume = $\frac{1}{3} \pi r^2 h$

SPHERES

A **sphere** is a 3D circular object that is rounded at all sides, and every point on the surface is equidistant from the center



Surface area = $4\pi r^2$

Volume = $\frac{4}{3} \pi r^3$

Practice Problems

G.13 Review

1. A cardboard box has dimensions of length = 3 cm, width = 5 cm, and height = 10 cm. What is the surface area?

a. 150 cm^2
b. 160 cm^2
c. 130 cm^2
d. 140 cm^2

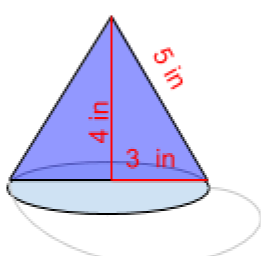
2. A soup can has dimensions diameter = 64 mm and height = 100 mm. What is the volume?

e. 320,000 mm^3
f. 400 cm^3
g. 120 cm^3
h. 200,000 mm^3

3. What is the approximate volume of a basketball with diameter 10 inches?

a. 450 in^3
b. 500 in^3
c. 520 in^3
d. 600 in^3

4. What is the surface area of this party hat?



a. 75 in^2
b. 65 in^2
c. 83 in^2
d. 60 in^2

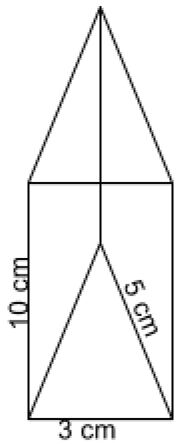
5. What is the surface area of a square pyramid with one side of the square = 1.5 m and slant height = 3 m?

a. 11 m^3
b. 11.25 m^3
c. 11.5 m^3
d. 11.75 ^{3}

6. What is the surface area of a tennis ball with diameter 3 inches?

a. 32 in^2
b. 30 in^2
c. 28 in^2
d. 25 in^2

7. What is the volume of this cardboard box?



a. 50 cm^3
b. 60 cm^3
c. 70 cm^3
d. 80 cm^3

8. What is the approximate volume of a square prism with side = 3.5 cm?

a. 43 cm^3
b. 46 cm^3
c. 50 cm^3
d. 55 cm^3

9. What is the volume of a regular triangular pyramid with area of base = 9 cm^2 and height = 5 cm?

a. 9 cm^3
b. 10 cm^3
c. 11 cm^3
d. 15 cm^3

Answer Key: Practice Problems

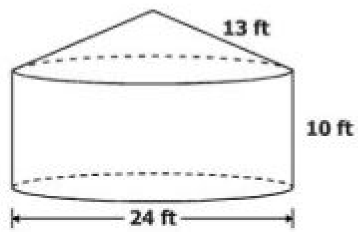
G.13 Geometry

1.	B
2.	A
3.	C
4.	A
5.	B
6.	C
7.	B
8.	A
9.	D

3D Object- Questions for Practice

SOL - Geometry

This container is composed of a right circular cylinder and a right circular cone.



Which is closest to the surface area of the container?

- A 490 ft^2
- B 754 ft^2
- C 1,243 ft^2
- D 1,696 ft^2

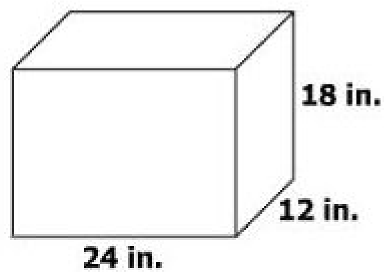
The height of a cylinder is 9.5 centimeters. The diameter of this cylinder is 1.5 centimeters longer than the height. Which is closest to the volume of the cylinder?

- A $1{,}150\pi$ cm^3
- B 287π cm^3
- C 165π cm^3
- D 105π cm^3

Which shape must have opposite sides that are parallel and congruent, and diagonals that are perpendicular bisectors of each other?

- **A** Parallelogram
- **B** Rectangle
- **C** Rhombus
- **D** Trapezoid

A rectangular prism is shown.



What is the surface area of the prism?

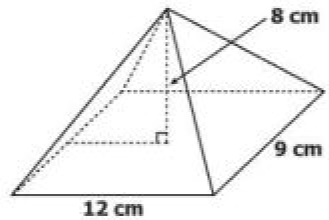
- **A** 156 sq in.
- **B** 936 sq in.
- **C** 1,872 sq in.
- **D** 5,184 sq in.

The ratio of the lengths of the radii of two spheres is 3:5. What is the ratio of the volumes of these two spheres?

☐ : ☐

1	3	5	9	15	25	27	125

A rectangular pyramid is shown.



What is the volume of the pyramid?

- **A** 864 cm^3
- **B** 432 cm^3
- **C** 288 cm^3
- **D** 108 cm^3