Math-No Calculator

1

Which of the following expressions is equal to 0 for some value of x ?

A) $|x-1|-1$

B) $|x+1|+1$

C) $|1-x|+1$

D) $|x-1|+1$

Answer: A

0-1=-1

The absolute value of -1=1

1-1=0

$\because |anything| \geqq 0$

$\therefore |anything| - 1$ can be 0. The minimum value for $|anything| + 1 = 0 + 1 = 1.$

Heart of Algebra

2

$$f(x) = \frac{3}{2}x + b$$

In the function above, b is a constant. If $f(6) = 7$, what is the value of $f(-2)$?

A) −5

B) −2

C) 1

D) 7

Answer: A

When $x = 6,$

$$f(x) = f(6) = \frac{3}{2} * 6 + b = 7$$

$$9 + b = 7$$

$$b = 7 - 9 = -2$$

When $x = -2,$

$$f(x) = f(-2) = \frac{3}{2} * (-2) + (-2)$$

$$-3 - 2 = -5$$

Heart of Algebra

3

$$\frac{x}{y} = 6$$

$$4(y + 1) = x$$

If (x, y) is the solution to the system of equations above, what is the value of y ?

A) 2

B) 4

C) 12

D) 24

Answer: A

$\because \frac{x}{y} = 6$

$\therefore x = 6y$

$\because 4(y + 1) = x$

$\therefore 4y + 4 = x = 6y$

$$4y + 4 - 4y = 6y - 4y$$

$$4 = 2y$$

$$y = 2$$

Heart of Algebra

4

If $f(x) = -2x + 5$, what is $f(-3x)$ equal to?

A) $-6x - 5$

B) $6x + 5$

C) $6x - 5$

D) $6x^2 - 15x$

Answer: B

To calculate $f(-3x)$, substitute x with -3x in the function $f(x) = -2x + 5$
$$f(-3x) = -2(-3x) + 5$$
$$= 6x + 5$$

Passport to Advanced Math

5

$$3(2x + 1)(4x + 1)$$

Which of the following is equivalent to the expression above?

A) $45x$

B) $24x^2 + 3$

C) $24x^2 + 18x + 3$

D) $18x^2 + 6$

Answer: C

$$(a + b)(c + d)$$
$$= ac + ad + bc + bd$$
$$3(2x + 1)(4x + 1)$$
$$= 3(2x * 4x + 2x * 1 + 1 * 4x + 1 * 1)$$
$$= 3(8x^2 + 6x + 1)$$
$$= 24x^2 + 18x + 3$$

Passport to Advanced Math

6

If $\dfrac{a - b}{b} = \dfrac{3}{7}$, which of the following must also be true?

A) $\dfrac{a}{b} = -\dfrac{4}{7}$

B) $\dfrac{a}{b} = \dfrac{10}{7}$

C) $\dfrac{a + b}{b} = \dfrac{10}{7}$

D) $\dfrac{a - 2b}{b} = -\dfrac{11}{7}$

Answer: B

To simplify the equation, cancel out denominators.
$$\frac{a - b}{b} = \frac{3}{7}$$
$$7b * \frac{a - b}{b} = \frac{3}{7} * 7b$$
$$7a - 7b = 3b$$
$$7a = 3b + 7b = 10b$$
$$\frac{a}{b} = \frac{10}{7}$$

Passport to Advanced Math

7

While preparing to run a marathon, Amelia created a training schedule in which the distance of her longest run every week increased by a constant amount. If Amelia's training schedule requires that her longest run in week 4 is a distance of 8 miles and her longest run in week 16 is a distance of 26 miles, which of the following best describes how the distance Amelia runs changes between week 4 and week 16 of her training schedule?

A) Amelia increases the distance of her longest run by 0.5 miles each week.

B) Amelia increases the distance of her longest run by 2 miles each week.

C) Amelia increases the distance of her longest run by 2 miles every 3 weeks.

D) Amelia increases the distance of her longest run by 1.5 miles each week.

Answer: D

(26-8)/(16-4) = 18/12 = 1.5

Heart of Algebra

8

Which of the following equations represents a line that is parallel to the line with equation $y = -3x + 4$?

A) $6x + 2y = 15$

B) $3x - y = 7$

C) $2x - 3y = 6$

D) $x + 3y = 1$

Answer: A

$y = ax + b, slope = a$
$y = -3x + 4, slope = -3$
For (A) $6x + 2y = 15$
$\qquad 2y = -6x + 15$
$\qquad\qquad y = -3x + \frac{15}{2} \qquad slope = -3$

When two lines are parallel, slope is equal.

9

$$\sqrt{x - a} = x - 4$$

If $a = 2$, what is the solution set of the equation above?

A) $\{3, 6\}$

B) $\{2\}$

C) $\{3\}$

D) $\{6\}$

Answer: D

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$$\sqrt{x-a} = x-4$$
$$\left(\sqrt{x-a}\right)^2 = (x-4)^2$$
$$x - a = x^2 - 8x + 16$$
$$\because a = 2$$
$$\therefore x^2 - 8x - x + 16 + 2 = 0$$
$$x^2 - 9x + 18 = 0$$
$$\because x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$\therefore x = \{3,6\}$$
$$\because \sqrt{x-a}$$
$$\therefore x - a \geq 0$$
$$x \geq 2$$
$$\therefore x = 6$$

Passport to Advanced Math

10

If $\dfrac{t+5}{t-5} = 10$, what is the value of t ?

A) $\dfrac{45}{11}$

B) 5

C) $\dfrac{11}{2}$

D) $\dfrac{55}{9}$

Answer: D

t+5=10(t-5)
t+5=10t-50
5+50=10t-t
　55=9t

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11

$$x = 2y + 5$$
$$y = (2x - 3)(x + 9)$$

How many ordered pairs (x, y) satisfy the system of equations shown above?

A) 0
B) 1
C) 2
D) Infinitely many

Answer: C

$x = 2y + 5$ (1)

$y = (2x - 3)(x + 9)$

$y = 2x^2 + 18x - 3x - 27$

$y = 2x^2 + 15x - 27$ (2)

Substitute (2) into (1)

$x = 2(2x^2 + 15x - 27) + 5$

$x = 4x^2 + 30x - 54 + 5$

$0 = 4x^2 + 29x - 49$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

2 solutions

Passport to Advanced Math

12

Ken and Paul each ordered a sandwich at a restaurant. The price of Ken's sandwich was x dollars, and the price of Paul's sandwich was \$1 more than the price of Ken's sandwich. If Ken and Paul split the cost of the sandwiches evenly and each paid a 20% tip, which of the following expressions represents the amount, in dollars, each of them paid? (Assume there is no sales tax.)

A) $0.2x + 0.2$

B) $0.5x + 0.1$

C) $1.2x + 0.6$

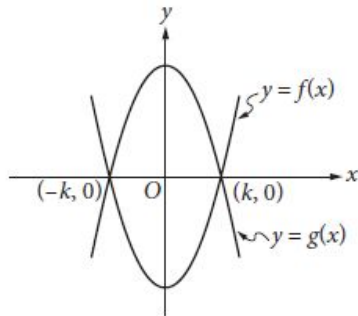D) $2.4x + 1.2$

Answer: C

Ken   x
Paul x+1
 (x+x+1)/2*(1+20%)
=(2x+1)/2*1.2
=(x+0.5)*1.2
=1.2x+0.6

Heart of Algebra

13

The functions f and g, defined by $f(x) = 8x^2 - 2$ and $g(x) = -8x^2 + 2$, are graphed in the xy-plane above. The graphs of f and g intersect at the points $(k,0)$ and $(-k,0)$. What is the value of k ?

A) $\dfrac{1}{4}$

B) $\dfrac{1}{2}$

C) 1

D) 2

Answer: B

At $(k, 0)$ and $(-k, 0)$, $f(x) = g(x) = 0$
$$8x^2 - 2 = 0$$
$$8x^2 = 2$$
$$x^2 = \frac{2}{8} = \frac{1}{4}$$
$$x = \pm\frac{1}{2}$$
$$k = \pm\frac{1}{2}$$

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14

$$\frac{8 - i}{3 - 2i}$$

If the expression above is rewritten in the form $a + bi$, where a and b are real numbers, what is the value of a ? (Note: $i = \sqrt{-1}$)

A) 2

B) $\dfrac{8}{3}$

C) 3

D) $\dfrac{11}{3}$

Answer: A

$\because i = \sqrt{-1}$

$\therefore i^2 = -1$

$\dfrac{8 - i}{3 - 2i} = a + bi$

$\dfrac{8 - i}{3 - 2i} * (3 - 2i) = (a + bi)(3 - 2i)$

$8 - i = 3a - 2ai + 3bi - 2bi^2$

$8 - i = (3a + 2b) - (2a - 3b)i$

$3a + 2b = 8 \quad (1)$

$2a - 3b = 1 \quad (2)$

(1)*3: $3(3a+2b) = 8*3$
$\qquad\qquad 9a+6b = 24 \quad (3)$

(2)*2: $2(2a-3b) = 1*2$
$\qquad\qquad 4a-6b = 2 \quad (4)$

(3) + (4): $9a+4a+6b-6b = 24+2$
$\qquad\qquad\qquad 13a = 26$
$\qquad\qquad\qquad a = 2$

15

$$x^2 - \frac{k}{2}x = 2p$$

In the quadratic equation above, k and p are constants. What are the solutions for x ?

A) $x = \frac{k}{4} \pm \frac{\sqrt{k^2 + 2p}}{4}$

B) $x = \frac{k}{4} \pm \frac{\sqrt{k^2 + 32p}}{4}$

C) $x = \frac{k}{2} \pm \frac{\sqrt{k^2 + 2p}}{2}$

D) $x = \frac{k}{2} \pm \frac{\sqrt{k^2 + 32p}}{4}$

Answer: B

$$x^2 - \frac{k}{2}x = 2p$$

$$x^2 - \frac{k}{2}x - 2p = 0$$

$$a = 1, b = -\frac{k}{2}, c = -2p$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

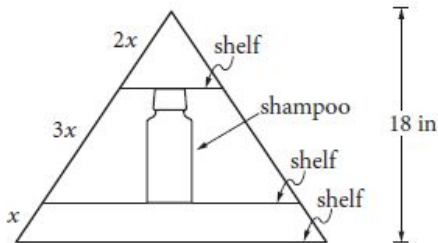$$\frac{-b}{2a} = \frac{\frac{k}{2} * 2}{2 * 2} = \frac{k}{4}$$

$$\sqrt{b^2 - 4ac} = \sqrt{\frac{k^2}{4} + 8p} = \sqrt{\frac{1}{4}(k^2 + 32p)}$$

$$= \frac{\sqrt{k^2 + 32p}}{2}$$

$$\frac{\sqrt{b^2 - 4ac}}{2a} = \frac{\sqrt{k^2 + 32p}}{2 * 2a} = \frac{\sqrt{k^2 + 32p}}{4}$$

$$x = \frac{k}{4} \pm \frac{\sqrt{k^2 + 32p}}{4}$$
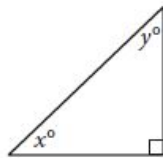
Passport to Advanced Math

16

Jim has a triangular shelf system that attaches to his showerhead. The total height of the system is 18 inches, and there are three parallel shelves as shown above. What is the maximum height, in inches, of a shampoo bottle that can stand upright on the middle shelf?
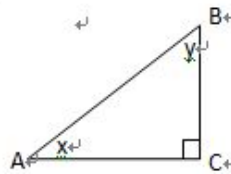
Answer: 9

x:3x:2x=1:3:2
18/(1+3+2)*3=9

17

In the triangle above, the sine of $x°$ is 0.6. What is the cosine of $y°$?

Answer: ⅗ or 0.6

$$\sin x = \frac{BC}{BA} = 0.6$$

$$\cos y = \frac{BC}{BA} = \sin x = 0.6$$

18

$$x^3 - 5x^2 + 2x - 10 = 0$$

For what real value of x is the equation above true?

Answer: 5

$$(x^3 - 5x^2) + (2x - 10) = 0$$
$$x^2(x - 5) + 2(x - 5) = 0$$
$$(x^2 - 2)(x - 5) = 0$$
$$x^2 - 2 = 0 \quad x - 5 = 0$$
$$x^2 = 2 \qquad \underline{x = 5}$$

$$\underline{x = +\sqrt{2}}$$

Passport to Advanced Math

19

$$-3x + 4y = 20$$
$$6x + 3y = 15$$

If (x, y) is the solution to the system of equations above, what is the value of x ?

Answer: 0

-3x+4y=20   (1)
6x+3y=15   (2)
-2*(1): -2(-3x+4y) = -2*20
                6x-8y = -40   (3)
(2) - (3): 11y=55
                y=5
Substitute y with 5 in (3)
6x-8*5 = -40
6x-40 = -40
6x = 0
x = 0

Heart of Algebra

20

The mesosphere is the layer of Earth's atmosphere between 50 kilometers and 85 kilometers above Earth's surface. At a distance of 50 kilometers from Earth's surface, the temperature in the mesosphere is −5° Celsius, and at a distance of 80 kilometers from Earth's surface, the temperature in the mesosphere is −80° Celsius. For every additional 10 kilometers from Earth's surface, the temperature in the mesosphere decreases by $k°$ Celsius, where k is a constant. What is the value of k ?

Answer: 25

(80-50)/10=3
-5-(-80)=75
75/3=25

Heart of Algebra

Additional Practice

1.	2.
Which of the following expressions is equal to 3 for some value of x? A) $\|2x\| + 5 = 3$ B) $\left\|\frac{5}{x}\right\| + 3 = 3$ C) $\|\|3 - x\| + 3\| + 1 = 3$ D) $\|x + 5\| + 1 = 3$ Answer: D	$$f(x) = \frac{4}{3}x - b$$ In the function above, b is a constant. If $f(6) = 16$, what is the value of $f(-15)$? A) -28 B) -12 C) 12 D) 28 Answer: B
3.	4.
$$\frac{x}{y} = 5$$ $$3(y - 13 + x) = x$$ If (x, y) is the solution to the system of equations above, what is the value of y? A) 19.5 B) $\frac{39}{7}$ C) 3 D) 6 Answer: C	If $f(x) = -3x - 4$, what is $f(-5x + 3)$ equal to? A) $15x - 1$ B) $15x - 13$ C) $15x^2 - 12$ D) $15x^2 + 5$ Answer: B

5.

$$4(3x + 4)(2x - 2)$$

Which of the following is equivalent to the expression above?

A) $24x^2 - 8$
B) $20x^2 - 32$
C) $24x^2 + 8x - 8$
D) $24x^2 + 8x - 32$

Answer: D

6.

If $\frac{a+b}{a-2b} = \frac{5}{11}$, which of the following must also be true?

A) $\frac{a}{b} = -\frac{1}{6}$
B) $\frac{a}{a-b} = -\frac{1}{3}$
C) $\frac{a}{b} = -\frac{7}{2}$
D) $\frac{a-b}{b} = -\frac{5}{2}$

Answer: C

7.

While preparing to run a marathon, Andrew created a training schedule in which the distance of his longest run every week increased by a constant amount. If Andrew's training schedule requires that his longest run in week 13 is a distance of 27 miles and his longest run in week 6 is a distance of 13 miles, which of the following best describes how the distance Andrew runs changes between week 13 and week 6 of his training schedule?

A) Andrew increases the distance of his longest run by 0.5 miles each week.
B) Andrew increases the distance of his longest run by 2 miles each week.
C) Andrew increases the distance of his longest run by 2 miles every 4 weeks.
D) Andrew increases the distance of his longest run by 1.5 miles each week.

Answer: B

8.

Which of the following equations represents a line that is parallel to the line with equation
$y = -2x - 5$?

A) $x + 2y = 4$
B) $2x - 4y = 1$
C) $4x + 2y = 1$
D) $4x - y = 2$

Answer: C