

Math-No Calculator

<p>1</p> <p>A painter will paint n walls with the same size and shape in a building using a specific brand of paint. The painter's fee can be calculated by the expression $nK\ell h$, where n is the number of walls, K is a constant with units of dollars per square foot, ℓ is the length of each wall in feet, and h is the height of each wall in feet. If the customer asks the painter to use a more expensive brand of paint, which of the factors in the expression would change?</p> <p>A) h B) ℓ C) K D) n</p>	<p>Answer: C</p> <p>This problem focuses on understanding what variables relate to in a word problem.</p> <p>The change being made is the cost of the paint, and cost is something measured in dollars (\$)</p> <p>This change should affect the variable that is also in terms of dollars, which would be K: "a constant with units of dollars per square foot"</p> <p>-- Andrew Bazak</p> <p>Passport to Advanced Math</p>
<p>2</p> <p>If $3r = 18$, what is the value of $6r + 3$?</p> <p>A) 6 B) 27 C) 36 D) 39</p>	<p>Answer: D</p> <p>First solve for r using the first equation: $3r = 18 \rightarrow r = 6$, divide each side by 3</p> <p>Plug in $r = 6$ into the given expression: $6r + 3 = (6)(6) + 3 = 36 + 3 = 39$</p> <p>-- Andrew Bazak</p> <p>Heart of Algebra</p>

3

Which of the following is equal to $a^{\frac{2}{3}}$, for all values of a ?

- A) $\sqrt{a^{\frac{1}{3}}}$
- B) $\sqrt{a^3}$
- C) $\sqrt[3]{a^{\frac{1}{2}}}$
- D) $\sqrt[3]{a^2}$

Answer: D

$\frac{2}{3}$ is basically $2 * \frac{1}{3}$, so it can be written as $a^2 * a^{1/3}$, and $a^{1/3} = \sqrt[3]{a}$

Therefore, $a^{2/3} = a^2 * a^{1/3} = \sqrt[3]{a^2}$

-- Wendy Wan

Passport to Advanced Math

4

The number of states that joined the United States between 1776 and 1849 is twice the number of states that joined between 1850 and 1900. If 30 states joined the United States between 1776 and 1849 and x states joined between 1850 and 1900, which of the following equations is true?

- A) $30x = 2$
- B) $2x = 30$
- C) $\frac{x}{2} = 30$
- D) $x + 30 = 2$

Answer: B

the number of states from 1776 to 1849 is twice the number of that between 1850-1900, and the number of that between 1850-1900 is x .

So the number between 1776-1849 should be $2x$ (because it is twice the number of 1850-1900)

The number between 1776-1849 is also 30, according to the third line of the question.

Therefore, $2x=30$.

--Wendy Wan

Heart of Algebra

5

If $\frac{5}{x} = \frac{15}{x+20}$, what is the value of $\frac{x}{5}$?

- A) 10
- B) 5
- C) 2
- D) $\frac{1}{2}$

Answer: C

$$\frac{5}{x} = \frac{15}{x+20}$$
$$5(x+20) = x \cdot 15$$
$$5x+100 = 15x$$
$$100 = 10x$$
$$x = 10$$

therefore,

$$x/5 = 10/5 = 2$$

-- Wendy Wan

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6

$$2x - 3y = -14$$
$$3x - 2y = -6$$

If (x, y) is a solution to the system of equations above, what is the value of $x - y$?

- A) -20
- B) -8
- C) -4
- D) 8

Answer: C

add both sides of the two equations together and get

$$5x - 5y = -20$$

then divide each side by 5

$$x - y = -4$$

-- Wendy Wan

Heart of Algebra

7

x	$f(x)$
0	3
2	1
4	0
5	-2

The function f is defined by a polynomial. Some values of x and $f(x)$ are shown in the table above. Which of the following must be a factor of $f(x)$?

- A) $x - 2$
- B) $x - 3$
- C) $x - 4$
- D) $x - 5$

Answer: C

According to the table we can see that one solution to $f(x) = 0$ is $x=4$

So when $x = 4$ if the factor equals 0, it must be a factor of $f(x)$

Among all the choices only C fulfill the condition above

Therefore, C

--Wendy Wan

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8

The line $y = kx + 4$, where k is a constant, is graphed in the xy -plane. If the line contains the point (c, d) , where $c \neq 0$ and $d \neq 0$, what is the slope of the line in terms of c and d ?

- A) $\frac{d-4}{c}$
- B) $\frac{c-4}{d}$
- C) $\frac{4-d}{c}$
- D) $\frac{4-c}{d}$

Answer: A

the slope of the liner is the value of k ,

plug in (c,d) into the given expression:

$$d = kc + 4$$

$$kc = d - 4$$

$$k = (d - 4) / c$$

--Wendy Wan

Heart of Algebra

9

$$kx - 3y = 4$$

$$4x - 5y = 7$$

In the system of equations above, k is a constant and x and y are variables. For what value of k will the system of equations have no solution?

- A) $\frac{12}{5}$
- B) $\frac{16}{7}$
- C) $-\frac{16}{7}$
- D) $-\frac{12}{5}$

Answer: A

from $4x - 5y = 7$ we can get

$$y = (4x-7)/5,$$

plug in $y = (4x-7)/5$ into the first expression:

$$kx - 3((4x-7)/5) = 4$$

$$kx - (12x-21)/5 = 4$$

$$kx - \frac{12}{5}x + \frac{21}{5} = 4$$

$$(k - 12/5)x = -\frac{1}{5}$$

$$(5k - 12)x = -1$$

$$(12-5k)x = 1$$

$$x = 1/(12 - 5k)$$

We know that denominator cannot be 0 so when $12 - 5k = 0$, the equations will have no solution

Therefore:

$$12 - 5k = 0$$

$$5k = 12$$

$$k = 12/5$$

-- Wendy Wan

Heart of Algebra

10

In the xy -plane, the parabola with equation $y = (x - 11)^2$ intersects the line with equation $y = 25$ at two points, A and B . What is the length of \overline{AB} ?

- A) 10
- B) 12
- C) 14
- D) 16

Answer: A

Plug in $y=25$:

$$(x-11)^2 = 25$$

Take square root of both sides:

$$x - 11 = \pm 5$$

If +5:

$$x - 11 = 5$$

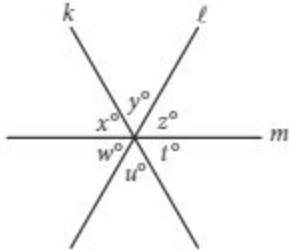
$$x = 16$$

If -5:

$$x - 11 = -5$$

$$x = 6$$

	<p>Points A and B: (6, 25) and (16, 25)</p> <p>Distance between A and B: = 16 - 6 = 10</p> <p>Rachel Goodgold Passport to Advance Math</p>
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<p>11</p>  <p>Note: Figure not drawn to scale.</p> <p>In the figure above, lines k, l, and m intersect at a point. If $x + y = u + w$, which of the following must be true?</p> <ul style="list-style-type: none"> I. $x = z$ II. $y = w$ III. $z = t$ <ul style="list-style-type: none"> A) I and II only B) I and III only C) II and III only D) I, II, and III 	<p>Answer: B</p> <p>We are given $x + y = u + w$</p> <p>Because k and l intersect m to form the angles, we have the following: $z = w$, $y = u$, and $x = t$ $w + u + t = 180$ $x + y + z = 180$</p> <p>so $w + u + t = x + y + z$ since $u + w = x + y$, $t = z$ meaning III is true</p> <p>above, however, we also determined that $x = t$, so $x = t = z$ and $x = z$ I is also true</p> <p>since $u + w = x + y$ and $y = u$ II would only be true if $w = u$ however, we have nothing that suggests those angles are equal. So the answer is B</p>
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12

$$y = a(x - 2)(x + 4)$$

In the quadratic equation above, a is a nonzero constant. The graph of the equation in the xy -plane is a parabola with vertex (c, d) . Which of the following is equal to d ?

- A) $-9a$
- B) $-8a$
- C) $-5a$
- D) $-2a$

Answer: A

The equation of a parabola is

$$y = a(x - c)^2 + d$$

$$y = a(x - 2)(x + 4)$$

$$y = a(x^2 + 4x - 2x - 8)$$

$$y = a(x^2 + 2x - 8)$$

$$y = a(x^2 + 2x + 1 - 9)$$

$$y = a[(x + 1)^2 - 9]$$

$$y = a(x + 1)^2 - 9a$$

$$\text{so } d = -9a$$

-written by Elise Favia
Passport to Advanced Math

13

The equation $\frac{24x^2 + 25x - 47}{ax - 2} = -8x - 3 - \frac{53}{ax - 2}$ is true for all values of $x \neq \frac{2}{a}$, where a is a constant.

What is the value of a ?

- A) -16
- B) -3
- C) 3
- D) 16

Answer: B

$$\frac{24x^2 + 25x - 47}{ax - 2} = -8x - 3 - \frac{53}{ax - 2}$$

$$\frac{24x^2 + 25x - 47}{ax - 2} = \frac{(-8x - 3)(ax - 2) - 53}{ax - 2}$$

$$24x^2 + 25x - 47 = -8ax^2 - 3ax + 16x + 6 - 53$$

$$24x^2 + 25x = -8ax^2 - 3ax + 16x$$

$$24x^2 + 9x = -8ax^2 - 3ax$$

$$24x^2 + 9x = a(-8x^2 - 3x)$$

$$\frac{24x^2 + 9x}{(-8x^2 - 3x)} = a$$

$$\frac{-3(-8x^2 - 3x)}{(-8x^2 - 3x)} = a$$

$$a = -3$$

-Liam Mulcahy
Passport to Advanced Math

14

What are the solutions to $3x^2 + 12x + 6 = 0$?

- A) $x = -2 \pm \sqrt{2}$
 B) $x = -2 \pm \frac{\sqrt{30}}{3}$
 C) $x = -6 \pm \sqrt{2}$
 D) $x = -6 \pm 6\sqrt{2}$

Answer: A

Use the quadratic formula!

If $ax^2+bx+c=0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-12 \pm \sqrt{12^2 - 4 \cdot 3 \cdot 6}}{2 \cdot 3}$$

$$x = \frac{-12 \pm \sqrt{144 - 72}}{6}$$

$$x = \frac{-12 \pm \sqrt{72}}{6}$$

$$x = \frac{-12 \pm 6\sqrt{2}}{6}$$

$$x = -2 \pm \sqrt{2}$$

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15

$$C = \frac{5}{9}(F - 32)$$

The equation above shows how a temperature F , measured in degrees Fahrenheit, relates to a temperature C , measured in degrees Celsius. Based on the equation, which of the following must be true?

- I. A temperature increase of 1 degree Fahrenheit is equivalent to a temperature increase of $\frac{5}{9}$ degree Celsius.
 II. A temperature increase of 1 degree Celsius is equivalent to a temperature increase of 1.8 degrees Fahrenheit.
 III. A temperature increase of $\frac{5}{9}$ degree Fahrenheit is equivalent to a temperature increase of 1 degree Celsius.
- A) I only
 B) II only
 C) III only
 D) I and II only

Answer: D

$$C = \frac{5}{9}(F - 32)$$

Calculate an increase of 1 degree F

$$C = \frac{5}{9}(32 - 32)$$

$$C = 0$$

$$C = \frac{5}{9}(33 - 32)$$

$$C = 1$$

So I is true

Calculate an increase of 1 degree C

$$0 = \frac{5}{9}(F - 32)$$

$$F = 32 \text{ (known from above)}$$

$$1 = \frac{5}{9}(F - 32)$$

$$\frac{9}{5} = F - 32$$

$$F = \frac{9}{5} + 32$$

$$F = \frac{32 \cdot 5 + 9}{5} = \frac{169}{5} = 33.8$$

$$\text{change} = 33.8 - 32 = 1.8$$

so II is true

Heart of Algebra

16

$$x^3(x^2 - 5) = -4x$$

If $x > 0$, what is one possible solution to the equation above?

Answer: 1 or 2

Expand out left side:

$$x^5 - 5x^3 = -4x$$

$$x^5 - 5x^3 + 4x = 0$$

Factor:

$$x(x^4 - 5x^2 + 4) = 0$$

$$x(x^2 - 4)(x^2 - 1) = 0$$

Solve for x:

$$x = 0$$

$$x^2 - 4 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

$$x^2 - 1 = 0$$

$$x^2 = 1$$

$$x = \pm 1$$

$$x = \{-2, -1, 0, 1, 2\}$$

But $x > 0$

$$\text{So } x = \{1, 2\}$$

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17

If $\frac{7}{9}x - \frac{4}{9}x = \frac{1}{4} + \frac{5}{12}$, what is the value of x ?

Answer: 2

Lowest common denominator is 36, so convert all fractions to be out of 36:

$$\left(\frac{28}{36}x\right) - \left(\frac{16}{36}x\right) = \left(\frac{9}{36}\right) + \left(\frac{15}{36}\right)$$

Multiply both sides by 36 to get rid of fractions:

$$28x - 16x = 9 + 15$$

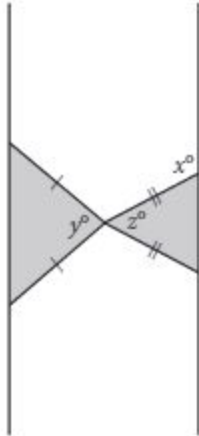
$$12x = 24$$

$$x = 2$$

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18



Note: Figure not drawn to scale.

Two isosceles triangles are shown above. If $180 - z = 2y$ and $y = 75$, what is the value of x ?

Answer: 105

We can see that

$$z + 2(180 - x) = 180$$

since the two triangles are isosceles

$$2(180 - x) = 180 - z$$

since $180 - z = 2y$

$$\text{we have } 2(180 - x) = 2y$$

$$y = 75$$

$$2(180 - x) = 2(75)$$

$$180 - x = 75$$

$$x = 180 - 75 = 105$$

-Written by Elise Favia

19

At a lunch stand, each hamburger has 50 more calories than each order of fries. If 2 hamburgers and 3 orders of fries have a total of 1700 calories, how many calories does a hamburger have?

Answer: 370

Let $x = \#$ of calories per hamburger

Let $y = \#$ of calories per order of fries

From the info given, can form the following equations:

$$x = 50 + y$$

$$2x + 3y = 1700$$

Substitute first equation for x in second equation:

$$2(50 + y) + 3y = 1700$$

Solve for y :

$$100 + 2y + 3y = 1700$$

$$5y + 100 = 1700$$

$$5y = 1600$$

$$y = 320$$

An order of fries has 320 calories

Solve for x :

$$x = 50 + y$$

$$x = 50 + 320$$

$$x = 370$$

A hamburger has 370 calories

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Heart of Algebra

20

In triangle ABC , the measure of $\angle B$ is 90° , $BC = 16$, and $AC = 20$. Triangle DEF is similar to triangle ABC , where vertices D , E , and F correspond to vertices A , B , and C , respectively, and each side of triangle DEF is $\frac{1}{3}$ the length of the corresponding side of triangle ABC . What is the value of $\sin F$?

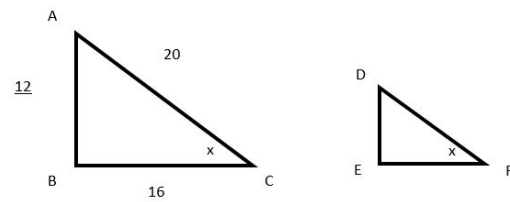
Answer: $\frac{3}{5}$ or 0.6

$$\sin x = \frac{\text{opposite}}{\text{hypotenuse}}$$

Since the triangles are similar the lengths of the side keep their proportions and the angles don't change.

Therefore $\sin c = \sin f$.

$$\sin x = \frac{12}{20} = \frac{3}{5} = 0.6$$



-Liam Mulcahy