

## Math-No Calculator

<p><b>1</b></p> <p>If <math>5x + 6 = 10</math>, what is the value of <math>10x + 3</math> ?</p> <p>A) 4 B) 9 C) 11 D) 20</p>	<p>Answer: C</p> <p><math>5x + 6 = 10</math>  <math>5x = 10 - 6</math>  <math>5x = 4</math>  <math>x = \frac{4}{5}</math>  <math>10 \left( \frac{4}{5} \right) + 3 = 11</math></p> <p style="text-align: right;">-Lauren St Laurent</p> <p>Heart of Algebra</p>
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<p><b>2</b></p> <p style="text-align: center;"> <math>x + y = 0</math>  <math>3x - 2y = 10</math> </p> <p>Which of the following ordered pairs <math>(x, y)</math> satisfies the system of equations above?</p> <p>A) <math>(3, -2)</math> B) <math>(2, -2)</math> C) <math>(-2, 2)</math> D) <math>(-2, -2)</math></p>	<p>Answer: B <math>x=2, y=-2</math></p> <p><math>x + y = 0</math>        substitute <math>2 + -2 = 0</math></p> <p><math>3x - 2y = 10</math>        substitute <math>= 3(2) - 2(-2) = 10</math></p> <p>no other options work when substituted</p> <p style="text-align: right;">-Lauren St Laurent</p> <p>Heart of Algebra</p>
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3

A landscaping company estimates the price of a job, in dollars, using the expression  $60 + 12nh$ , where  $n$  is the number of landscapers who will be working and  $h$  is the total number of hours the job will take using  $n$  landscapers. Which of the following is the best interpretation of the number 12 in the expression?

- A) The company charges \$12 per hour for each landscaper.
- B) A minimum of 12 landscapers will work on each job.
- C) The price of every job increases by \$12 every hour.
- D) Each landscaper works 12 hours a day.

Answer: A

Write out what you know!

$$60 + 12nh = \text{Total Cost}$$

$n$  = total landscapers

$h$  = total hours working per worker

If we know that the equation solves for the total cost. A is correct because the price of the job will rely on how many workers ( $n$ ), how many hours ( $h$ ), and how much it costs to hire that amount of workers per hour.

B is covered by  $n$

C would make sense only if it was addition (not multiplication)

D is covered by  $h$

-Lauren St Laurent

Heart of Algebra

4

$$9a^4 + 12a^2b^2 + 4b^4$$

Which of the following is equivalent to the expression shown above?

- A)  $(3a^2 + 2b^2)^2$
- B)  $(3a + 2b)^4$
- C)  $(9a^2 + 4b^2)^2$
- D)  $(9a + 4b)^4$

Answer: A

FOIL = multiply first, Outer, Inner, Last  
 $(3a^2 + 2b^2)^2$

multiply together:  $(3a^2 + 2b^2)(3a^2 + 2b^2)$

first:  $3a^2 \cdot 3a^2 = (9a^4)$

outer:  $3a^2 \cdot 2b^2 = 6a^2b^2$

inner:  $2b^2 \cdot 3a^2 = 6a^2b^2$

last:  $2b^2 \cdot 2b^2 = 4b^4$

Put together:  $(9a^4) + (6a^2b^2 + 6a^2b^2) + 4b^4$

Because the middle bracket has like terms you can add them together

$$9a^4 + 12a^2b^2 + 4b^4$$

-Lauren St Laurent

Passport to Advanced Math

5

$$\sqrt{2k^2 + 17} - x = 0$$

If  $k > 0$  and  $x = 7$  in the equation above, what is the value of  $k$ ?

- A) 2
- B) 3
- C) 4
- D) 5

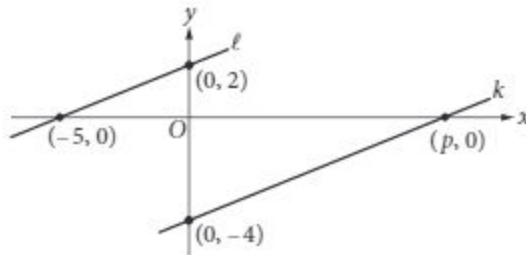
Answer: C

$$\begin{aligned} \sqrt{2k^2 + 17} - x &= 0 \\ \sqrt{2k^2 + 17} &= x \\ \sqrt{2k^2 + 17}^2 &= x^2 \\ 2k^2 + 17 &= x^2 \\ 2k^2 &= x^2 - 17 \\ 2k^2 &= (7)^2 - 17 \\ 2k^2 &= 49 - 17 \\ 2k^2 &= 32 \\ \frac{2k^2}{2} &= \frac{32}{2} \\ k^2 &= 16 \\ k &= 4 \end{aligned}$$

-Lauren St Laurent

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6



In the  $xy$ -plane above, line  $\ell$  is parallel to line  $k$ . What is the value of  $p$ ?

- A) 4
- B) 5
- C) 8
- D) 10

Answer: D

-Lauren St Laurent

Coordinates are given  $(x, y)$   
Parallel Lines have like slopes  
Slope =  $(y_2 - y_1) / (x_2 - x_1)$  of a line

We know all four points of the top line  
Slope =  $(2 - 0) / (0 - -5) = 2/5$

Set that slope equal to the slope of the bottom line

$$\frac{2}{5} = \frac{0 - -4}{p - 0}$$

$$\frac{2}{5} = \frac{4}{p}$$

Cross multiply

$$2p = 4 * 5$$

$$2p = 20$$

$$P = 20 / 2$$

$$P = 10$$

Heart of Algebra

7

If  $\frac{x^{a^2}}{x^{b^2}} = x^{16}$ ,  $x > 1$ , and  $a + b = 2$ , what is the value

of  $a - b$  ?

- A) 8
- B) 14
- C) 16
- D) 18

Answer: A

$\frac{x^{a^2}}{x^{b^2}} = x^{16}$ $a^2 - b^2 = 16$ $a^2 - b^2 = 4^2$ $a + b = 2$	<p>We know that:</p> $a^2 - b^2 = (a + b)x(a - b)$ <p>So:</p> $16 = 2(a - b)$ <p>Therefore:</p> $a - b = 8$
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-Liam Mulcahy

Passport to Advanced Math

8

$$nA = 360$$

The measure  $A$ , in degrees, of an exterior angle of a regular polygon is related to the number of sides,  $n$ , of the polygon by the formula above. If the measure of an exterior angle of a regular polygon is greater than  $50^\circ$ , what is the greatest number of sides it can have?

- A) 5
- B) 6
- C) 7
- D) 8

Answer: C

We know that exterior angles always add up to 360 degrees as shown by the problem.

$$7 \times 50 = 350 \quad 8 \times 50 = 400$$

Since  $350 < 360$ , the answer must be 7

-Liam Mulcahy

Heart of Algebra

9

The graph of a line in the  $xy$ -plane has slope 2 and contains the point  $(1, 8)$ . The graph of a second line passes through the points  $(1, 2)$  and  $(2, 1)$ . If the two lines intersect at the point  $(a, b)$ , what is the value of  $a + b$  ?

- A) 4
- B) 3
- C) -1
- D) -4

Answer: B

<p>Slope = 2 Point: <math>(1, 8)</math></p> $y - 8 = 2(x - 1)$ $y = 2x + 6$	<p>Point <math>(1, 2)</math> <math>(2, 1)</math></p> $\text{Slope}(m) = \frac{2 - 1}{1 - 2} = \frac{1}{-1} = -1$ <p>Using Point <math>(1, 2)</math></p> $y - 2 = -1(x - 1)$ $y = -x + 3$
<p>Setting the two equations equal</p> $-x + 3 = 2x + 6$ $-3 = 3x$ $x = -1$ <p>Plugging the value in to either equation</p> $y = 1 + 3 = 4$ $(a, b) = (-1, 4)$ $a + b = -1 + 4 = 3$	

-Liam Mulcahy

## Heart of Algebra

**10**

Which of the following equations has a graph in the  $xy$ -plane for which  $y$  is always greater than or equal to  $-1$  ?

- A)  $y = |x| - 2$   
 B)  $y = x^2 - 2$   
 C)  $y = (x - 2)^2$   
 D)  $y = x^3 - 2$

Answer: C

C is the correct answer. Any number multiplied by itself will always equal a positive number. In answer C, the exponent “2” means that the value in parentheses will be multiplied by itself

For example:  $(-2)^2 = (-2)(-2) = 4$   
 or  $(2)^2 = (2)(2) = 4$

-Lauren St.Laurent

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**11**

Which of the following complex numbers is equivalent to  $\frac{3 - 5i}{8 + 2i}$  ? (Note:  $i = \sqrt{-1}$ )

- A)  $\frac{3}{8} - \frac{5i}{2}$   
 B)  $\frac{3}{8} + \frac{5i}{2}$   
 C)  $\frac{7}{34} - \frac{23i}{34}$   
 D)  $\frac{7}{34} + \frac{23i}{34}$

Answer: C

$$\frac{3 - 5i}{8 + 2i} \cdot \frac{8 - 2i}{8 - 2i} = \frac{24 - 6i - 40i - 10}{64 + 4} = \frac{14 - 46i}{68}$$

$$= \frac{7 - 23i}{34}$$

-Liam Mulcahy

12

$$R = \frac{F}{N + F}$$

A website uses the formula above to calculate a seller's rating,  $R$ , based on the number of favorable reviews,  $F$ , and unfavorable reviews,  $N$ . Which of the following expresses the number of favorable reviews in terms of the other variables?

A)  $F = \frac{RN}{R - 1}$

B)  $F = \frac{RN}{1 - R}$

C)  $F = \frac{N}{1 - R}$

D)  $F = \frac{N}{R - 1}$

Answer: B

$$R = \frac{F}{N + F}$$

$$(RN + RF) = F$$

$$RN = F - RF = F(1 - R)$$

$$F = \frac{RN}{1 - R}$$

-Liam Mulcahy

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13

What is the sum of all values of  $m$  that satisfy  $2m^2 - 16m + 8 = 0$  ?

A)  $-8$

B)  $-4\sqrt{3}$

C)  $4\sqrt{3}$

D)  $8$

Answer: D

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-(-16) \pm \sqrt{16^2 - 4(8)(2)}}{2(2)}$$

$$\frac{16 \pm \sqrt{16^2 - 4(16)}}{4}$$

$$\frac{16 \pm \sqrt{16(12)}}{4}$$

$$4 \pm 4\sqrt{12}$$

Sum of all values = 8

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**14**

A radioactive substance decays at an annual rate of 13 percent. If the initial amount of the substance is 325 grams, which of the following functions  $f$  models the remaining amount of the substance, in grams,  $t$  years later?

- A)  $f(t) = 325(0.87)^t$
- B)  $f(t) = 325(0.13)^t$
- C)  $f(t) = 0.87(325)^t$
- D)  $f(t) = 0.13(325)^t$

Answer: A

You should know rate of decay =  $N(r)^t$

N= Initial amount

R= rate

t= time

r= 13% decrease ! so r = 0.87

t=t

N= 325

so A is correct

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**15**

The expression  $\frac{5x-2}{x+3}$  is equivalent to which of the following?

- A)  $\frac{5-2}{3}$
- B)  $5 - \frac{2}{3}$
- C)  $5 - \frac{2}{x+3}$
- D)  $5 - \frac{17}{x+3}$

Answer: D

I would first plug in a random value for "x".

Keep this consistent

For example, let's say  $x = 1$

$$(5(1) - 2) / (1 + 3) = \frac{3}{4}$$

If plug  $x=1$  into solution D

$$5 - (17) / (1 + 3) = \frac{3}{4}$$

They're the same!

-Lauren St Laurent

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**16**

The sales manager of a company awarded a total of \$3000 in bonuses to the most productive salespeople. The bonuses were awarded in amounts of \$250 or \$750. If at least one \$250 bonus and at least one \$750 bonus were awarded, what is one possible number of \$250 bonuses awarded?

Answer: 3, 6, or 9

$$\text{Total} = \$3000$$

If at least one \$250 and one \$750 bonus,

$$\text{Then } 3000 - 750 - 250 = 2000$$

All \$250 Bonuses	1 \$750, the rest \$750	2 \$750, the rest \$750
\$2000	\$2000-750=1250	\$2000-2(750)=500
$\frac{2000}{250} = 8$	$\frac{1250}{250} = 5$	$\frac{500}{250} = 2$
Total=9	Total=6	Total=3

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Heart of Algebra

17

$$2x(3x + 5) + 3(3x + 5) = ax^2 + bx + c$$

In the equation above,  $a$ ,  $b$ , and  $c$  are constants. If the equation is true for all values of  $x$ , what is the value of  $b$ ?

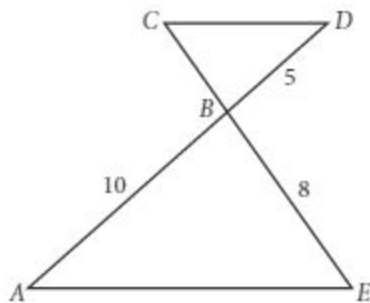
Answer: 19

$$\begin{aligned} &2x(3x + 5) + 3(3x + 5) \\ &= (6x^2 + 10x) + (9x + 15) \\ &= (6x^2 + 19x + 15) \\ &\text{So } b = 19 \end{aligned}$$

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18



In the figure above,  $\overline{AC} \parallel \overline{DE}$  and segment  $\overline{AD}$  intersects segment  $\overline{CE}$  at  $B$ . What is the length of segment  $\overline{CE}$ ?

Answer: 12

Shapes are in proportion to each other!

$$5/10 = CB/8$$

Cross multiply

$$5(8) = 10(CB)$$

$$40 = 10CB$$

$$40/10 = CB$$

$$CB = 4$$

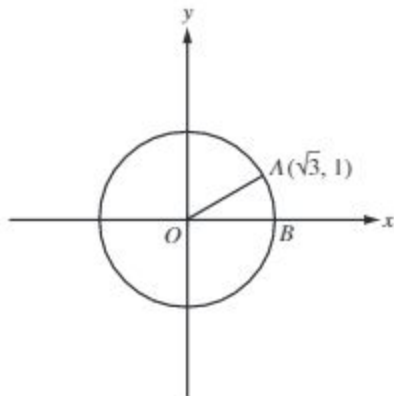
$$CB + BE = CE$$

$$4 + 8 = 12$$

-Lauren St Laurent



19



In the  $xy$ -plane above,  $O$  is the center of the circle, and the measure of  $\angle AOB$  is  $\frac{\pi}{a}$  radians. What is the value of  $a$ ?

Answer: 6

From the Unit Circle we know that at  $\frac{\pi}{6}$

Coordinates on the Unit Circle are  $(\frac{\sqrt{3}}{2}, \frac{1}{2})$

If radius is doubled  $(\sqrt{3}, 1)$

Answer: 6

-Liam Mulcahy

20

$$\begin{aligned} ax + by &= 12 \\ 2x + 8y &= 60 \end{aligned}$$

In the system of equations above,  $a$  and  $b$  are constants. If the system has infinitely many solutions, what is the value of  $\frac{a}{b}$ ?

Answer:  $\frac{1}{4}$  or 0.25

For this to have infinite solutions Equation 1 must be a multiple of Equation 2. Therefore the ratios of the constants must be equal.

$$\text{So: } \frac{a}{b} = \frac{2}{8} = \frac{1}{4}$$

-Liam Mulcahy

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